ME 314 - Engineering Design : Mechanical Components
Lecture 2

2.2 The Statistical Nature of Material Properties

Uncertainties are due to:

*** Variations in properties from place to place within a body**

*** Composion of material & the effect of variation in material properties**

*** Intensity & distribution of loading**

*** Intensity of stress concentrations, etc.**

In the Reliability Method of Design material properties, etc. are treated as stochastic or random variables.

Most material properties vary according to some distribution about their average or mean values.

The most likely distribution of a set of random data is the Gaussian or normal distribution.

The normal distribution which is defined in terms of two parameters:

The arithmetic mean, μ, and the standard deviation, S_d,

has the form

where x is the material property, f(x) is the frequency of occurrence of x in the population, and

Plots of (2.9a) are as shown

The mean μ is the most frequently occurring value of x.

we can expect to find

- 68% of population within $x = \mu \pm S_d$
- 95% of population within $x = \mu \pm 2S_d$
- 99% of population within $x = \mu \pm 3S_d$

Hence, the mean value, μ, alone is not a good predictor of the value of the variable x.

For example, suppose x = strength. There is a 50% chance that the samples of any material that you buy will have a strength less than the material's published mean value.

However, if S_d is published along with μ, we can factor down the strength value so that it is predictive of a larger percentage of population.

For steel, if S_d is not available, some reports indicate that $S_d = 0.08\mu$ is a reasonable assumption:

Note that in the tables of material properties given in text unless specified, "minimum" values are given and their statistical distributions are not of concern.

Reading Assignment: Chapter 2

Chapter 3: Load Determination

Basic concepts of Statics & dynamics (equilibrium & motion), impact forces, & beam loading are reviewed in this chapter.

Newton's first law: A body at rest will remain at rest and a body in motion with constant velocity will maintain that velocity unless acted upon by an external force.

Newton's second law: The time rate of change of linear {angular} momentum of a body is proportional to the resultant force {moment} acting on the body.

Angular momentum and the resultant moment are calculated about the same fixed point.

$$\Sigma F = ma\,, \qquad \Sigma M_G = d(H_G)/dt \qquad (3.1a)$$

where m is mass and G means the center of gravity. Choosing a cartesian coordinate system, Gxyz:

$$\Sigma F_x = ma_x\,, \quad \Sigma F_y = ma_y\,, \quad \Sigma F_z = ma_z \qquad (3.1b)$$

If x-, y-, and z- axes coincide with the principal axes of inertia of the body

$$H_G = I_x\,\omega_x\,\mathbf{i} + I_y\,\omega_y\,\mathbf{j} + I_z\,\omega_z\,\mathbf{k} \qquad (3.1c)$$

where I_x, I_y, and I_z are the principal mass moments of inertia about the principal axes. The angular momentum equations will then take the form

$$\begin{aligned}
\Sigma M_x &= I_x\,\alpha_x - (I_y - I_z)\,\omega_y\,\omega_z \\
\Sigma M_y &= I_y\,\alpha_y - (I_z - I_x)\,\omega_z\,\omega_x \qquad (3.1d)\\
\Sigma M_z &= I_z\,\alpha_z - (I_x - I_y)\,\omega_x\,\omega_y
\end{aligned}$$

where α_x, α_y, and α_z are the angular accelerations about the axes. I_x, I_y, and I_z are assumed to remain constant.

Newton's third law: The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense.

We use these three laws (and if necessary, kinematic relations) to analyze the forces that act on mechanical components. Some special cases are as follows:

Two-dimensional case:

$$\Sigma F_x = ma_x , \quad \Sigma F_y = ma_y , \quad \Sigma M_G = I_z \alpha_z$$

Static equilibrium:

$$\Sigma F_x = 0 , \quad \Sigma F_y = 0 , \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0 , \quad \Sigma M_y = 0 , \quad \Sigma M_z = 0$$

For equilibrium in 2D:

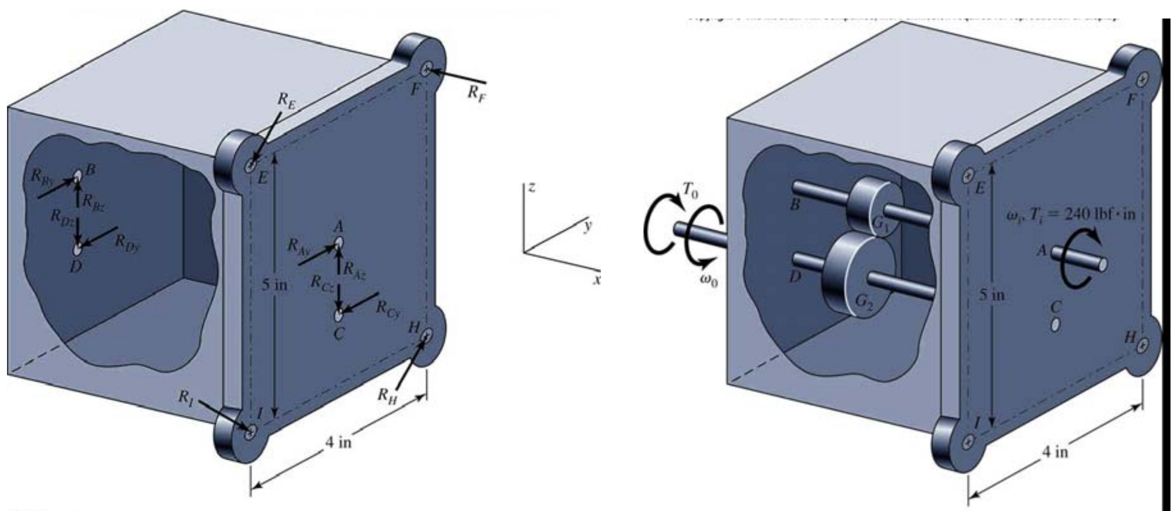
$$\Sigma F_x = 0 , \quad \Sigma F_y = 0 , \quad \Sigma M_z = 0$$

Free Body Diagram (FBD): The analysis of complex machines can be simplified by isolating each element and studying it by using FBD's. The key to success in breaking a complicated problem into manageable parts is to show all forces (external & internal).

Example: The input & output shafts *AB* and *CD* of the gear reducer shown are rotating at constant speeds ω_i and ω_o. The input & output torques are T_i = 240 lb-in and T_o. Shafts are supported by bearings at *A*, *B*,*C*, and *D*. G_1 and G_2 are spur gears with pitch radii of r_1 = 0.75" and r_2 = 1.5", respectively, and a pressure angle of φ = 20°.

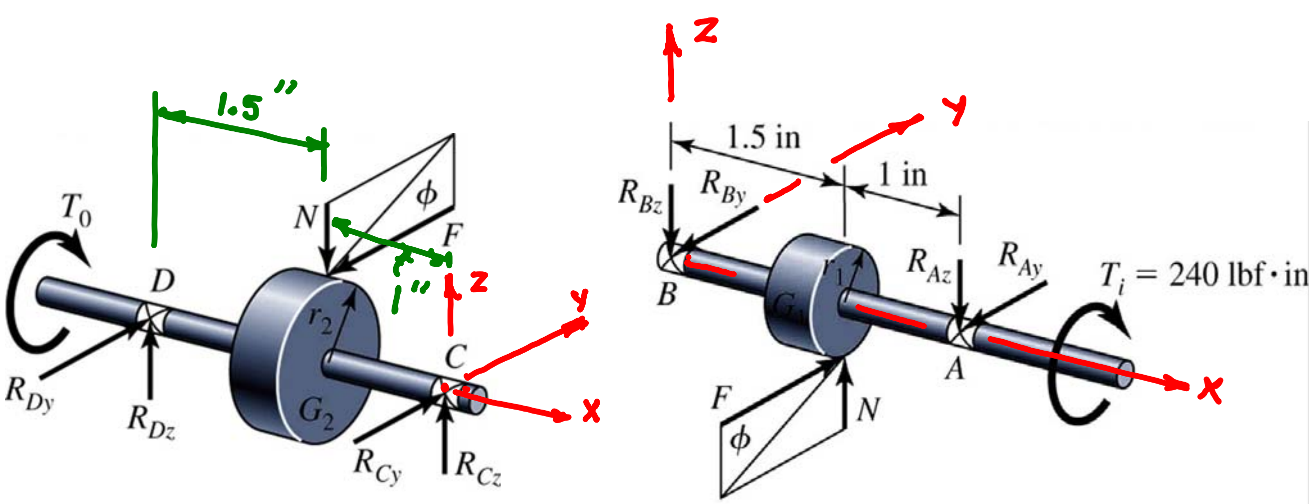
Draw the FBD of each member & find the reaction forces & moments at all points.

Assumption: Shafts are simply supported (friction is negligible). The weight of members are negligible. The mounting bolts *E*, *F*, *H*, and *I* are of the same size.



(*b*) Gear box

(*a*) Gear reducer



(*d*) Output shaft $r_2 = 1.5$ in

(*c*) Input shaft $r_1 = 0.75$ in

$\boxed{\text{shaft AB}}$

$\boxed{\text{Shaft CD}}$ $\Sigma M_x = 320(1.5) - T_o = 0 \Rightarrow T_o = 480 \text{ lbf.in}$

$$\Sigma M_y = -(116.5)(1) + R_{Dz}(2.5) = 0 \Rightarrow R_{Dz} = 46.6 \text{ lbf}$$

$$\Sigma M_z = (320)(1) - R_{Dy}(2.5) = 0 \Rightarrow R_{Dy} = 128 \text{ lbf}$$

$$\Sigma F_x \equiv 0, \quad \Sigma F_y = R_{Cy} + 128 - 320 = 0 \Rightarrow R_{Cy} = 192 \text{ lbf}$$

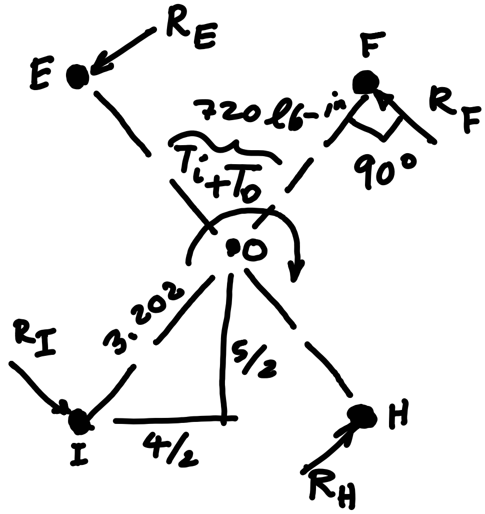
$$\Sigma F_z = R_{Cz} + 46.6 - 116.5 = 0 \Rightarrow R_{Cz} = 69.9 \text{ lbf}$$

Note from Fig. (b) that the sum of all bearing reactions is zero while the net moment about the x-axis by R_{Ay}, R_{Cy}, R_{By}, and R_{Dy}, is

$$192(2.25) + 128(2.25) = 720 \text{ lbf. in}$$

This is the same as $T_i + T_o = 240 + 480 = 720$ lbf. in

The gear box tends to rotate about the x-axis because of the net torque 720 lbf. in (note that T_o and ω_o are opposite because T_o is the resistive load on the system opposing the motion ω_o). The bolt forces R_E, R_F, R_H, & R_I provide 720 lbf.in in the opposite sense.



Homework due next Tuesday (see "Assignments" on Blackboard):

1. Problem Set 1
2. Problem Set 2

* Redo Case Study 3A (Automobile Scissors-Jack Loading Analysis) for a force of P = 500 lb. You can solve the equations by hand or software (e.g. the program Matrix mentioned in text).

* Redo Case Study 5A (Fourbar Linkage Loading Analysis) filling in all details. In particular, the kinematic analysis (i.e., finding linear & angular accelerations) which is missing in text.

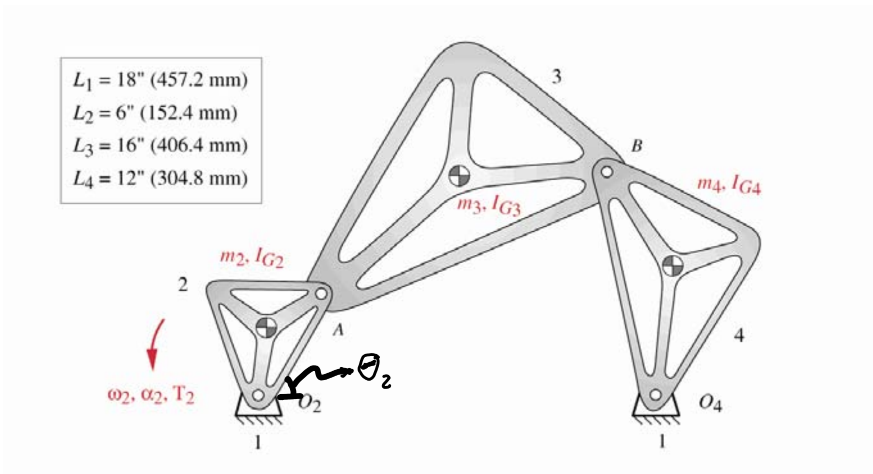


Figure 3-12
Fourbar Linkage Schematic and Basic Dimensions (See Table 3-6 for more information).

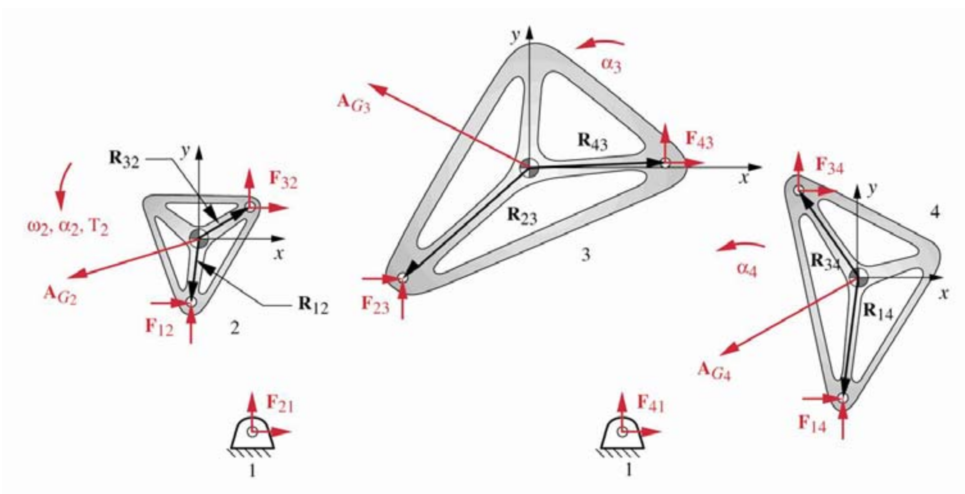


Figure 3-13
Free-Body Diagrams of Elements in a Fourbar Linkage.

Variable	Value	Unit
θ_2	30.00	deg
ω_2	120.00	rpm
$mass_2$	0.525	kg
$mass_3$	1.050	kg
$mass_4$	1.050	kg
I_{cg2}	0.057	kg-m^2
I_{cg3}	0.011	kg-m^2
I_{cg4}	0.455	kg-m^2
R_{12x}	–46.9	mm
R_{12y}	–71.3	mm
R_{32x}	85.1	mm
R_{32y}	4.9	mm
R_{23x}	–150.7	mm
R_{23y}	–177.6	mm
R_{43x}	185.5	mm
R_{43y}	50.8	mm
R_{14x}	–21.5	mm
R_{14y}	–100.6	mm
R_{34x}	–10.6	mm
R_{34y}	204.0	mm

Table 3-6 - part 1
Case Study 5A Given and Assumed Data.

Variable	Value	Unit
F_{12x}	–255.8	N
F_{12y}	–178.1	N
F_{32x}	252.0	N
F_{32y}	172.2	N
F_{34x}	–215.6	N
F_{34y}	–163.9	N
F_{14x}	201.0	N
F_{14y}	167.0	N
F_{43x}	215.6	N
F_{43y}	163.9	N
F_{23x}	–252.0	N
F_{23y}	–172.2	N
T_{12}	–3.55	N-m
α_3	56.7	rad/sec^2
α_4	138.0	rad/sec^2
A_{cg2x}	–7.4	rad/sec^2
A_{cg2y}	–11.3	rad/sec^2
A_{cg3x}	–34.6	rad/sec^2
A_{cg3y}	–7.9	rad/sec^2
A_{cg4x}	–13.9	rad/sec^2
A_{cg4y}	2.9	rad/sec^2

Table 3-6 - part 2
Case Study 5A Calculated Data.

3.7 Vibrations

The motion of a body which oscillates about a position of equilibrium is referred to as vibration.

Most vibrations in machines are undesirable because they are accompanied with increased stresses and energy losses.

Vibration occurs when the applied load is dynamic (i.e., changes with time) and material has elasticity which is practically all materials. If the element is infinitely stiff (i.e., rigid), then vibration will not occur.

We must consider vibration on a machine when loads are dynamic.

The procedure is to find the "natural frequency" of the machine in order to predict and avoid an undesirable condition called "resonance."

Resonance is a condition that can be experienced if the operating frequency applied to the system is the same as any one of its natural frequencies. Any real system can have an infinite number of natural frequencies.

At a minimum, we should find the system's lowest or fundamental natural frequency which creates the largest magnitude of vibration.

For complete analysis, Finite Element Analysis (FEA) is used.

Example: Cam-Follower System

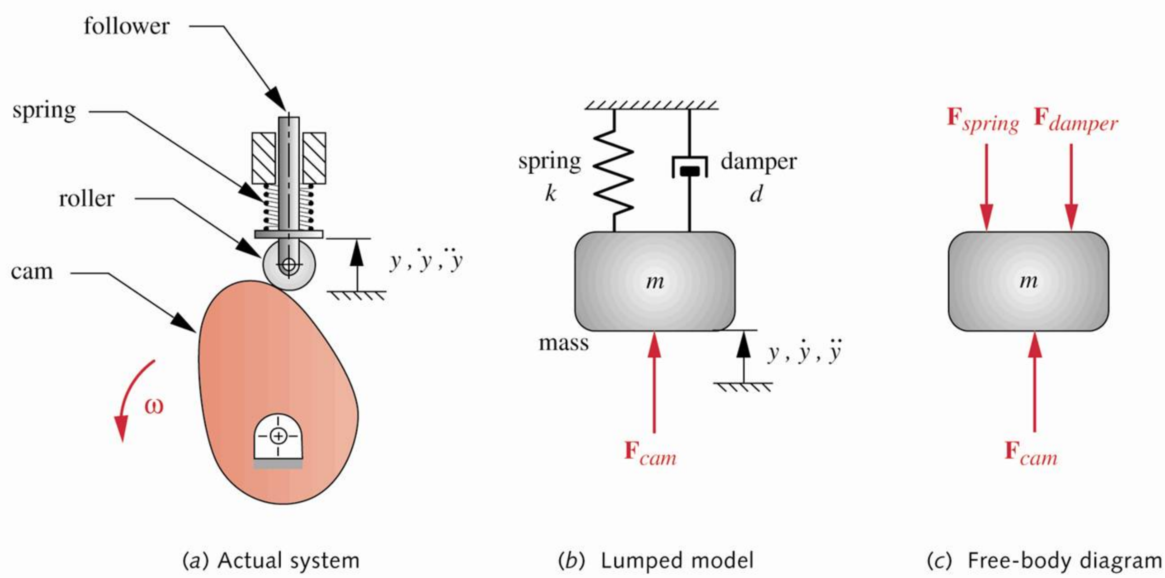


Figure 3-15
Lumped Model of a Cam-Follower Dynamic System.

"If the input angular velocity applied to a rotating system is close to ω_n or ω_d, the vibratory response will be very large. This can create large forces and cause failure. Thus it is necessary to avoid operation at or near the natural frequencies if possible."